$$y = a\sin(k(x-d)) + c \qquad \text{and} \qquad y = a\cos(k(x-d)) + c$$

vert. stretch (amplitude)

horiz. stretch

horiz. shift (phase shift)

vert. shift (axis of curve)

STEPS

1. Determine the Horizontal Stretch by finding the period of the curve. * period $= \dfrac{2\pi}{|k|}$
2. Determine the number of radians between the keys points $\left(i.e.\ \dfrac{\text{period}}{4}\right)$.
3. Using the phase shift and number of radians between key points, create a scale for the *x*-axis based on the period of the curve (usually 12 boxes = period).
4. Sketch the axis of the curve (midline), and, lines showing where the max and min values are located.

   amplitude $= |a|$ (cannot be negative)

   $= \dfrac{\text{max} - \text{min}}{2}$
5. Remember to consider reflections.

Example 1: Determine the amplitude, period, phase shift, and vertical translation for each function, with respect to the base function. Sketch one cycle for each of the following functions, angles are in radians.

a) $y = 3\sin\dfrac{1}{2}\left(x + \dfrac{\pi}{4}\right)$

amp: 3

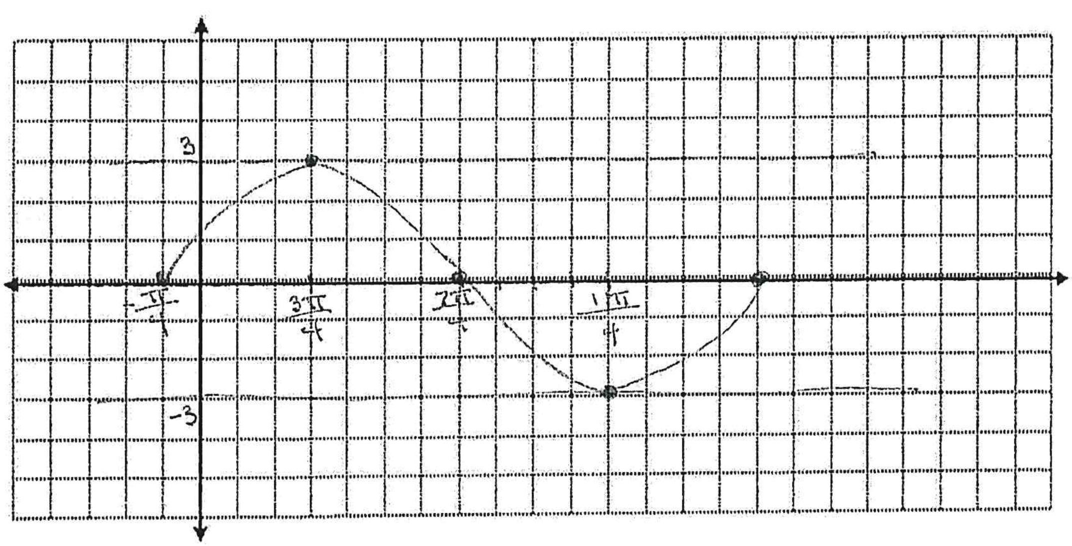
phase shift: left $\dfrac{\pi}{4}$

vert. shift: none

period: $\dfrac{2\pi}{\frac{1}{2}}$

$= 4\pi$

key points located every $\dfrac{4\pi}{4} = \pi$ units



b) $y=\frac{1}{2}\cos(3x)+1$

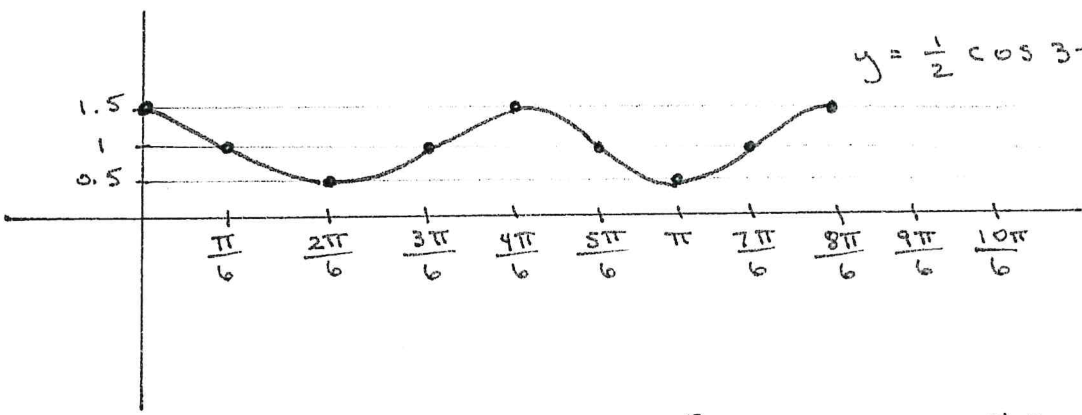
period: $\frac{2\pi}{3}$

amp: $\frac{1}{2}$

p.s.: none

vert. shift: up 1
(axis of curve)

key points: $\frac{\frac{2\pi}{3}}{4} = \frac{2\pi}{3}\cdot\frac{1}{\cancel{4}\,2} = \frac{\pi}{6}$



** (see note below)

c) $y=-\sin 2\left(x-\frac{\pi}{3}\right)$
↑
reflection

phase shift: $\frac{\pi}{3}$ right $\therefore$ "1st" point: $+\frac{\pi}{3}$

no vert. shift

amp.: 1

period: $\frac{2\pi}{2}=\pi$

key points: $\frac{\pi}{4}$

"2nd" pt: $\frac{\pi}{3}+\frac{\pi}{4}=\frac{7\pi}{12}$

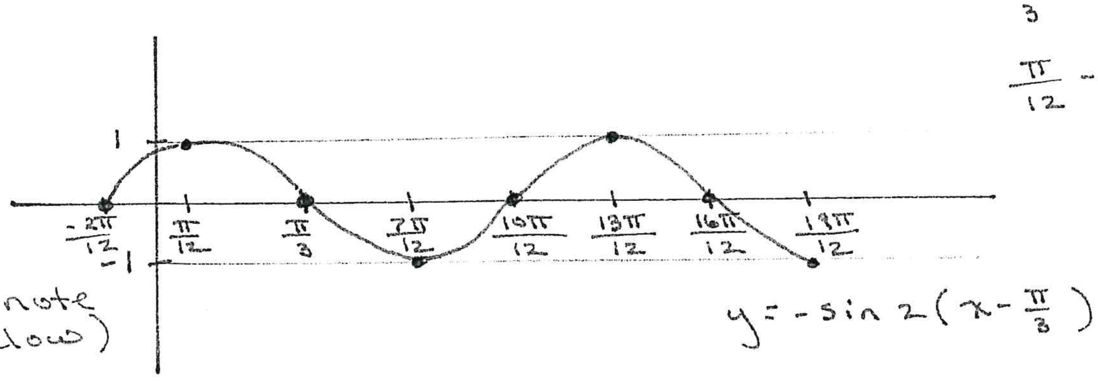
"3rd" pt: $\frac{7\pi}{12}+\frac{\pi}{4}=\frac{10\pi}{12}$

"4th" pt: $\frac{10\pi}{12}+\frac{\pi}{4}=\frac{13\pi}{12}$

Backwards from 1st point:

$\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$ and

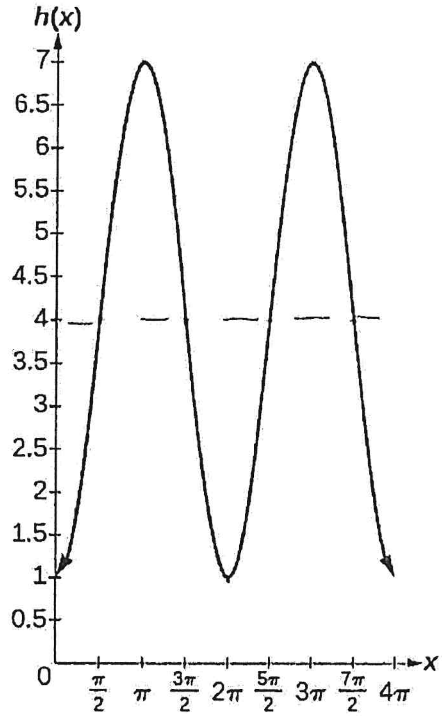
$\frac{\pi}{12}-\frac{\pi}{4}=-\frac{2\pi}{12}$



** (see note below)

** Note: all values on x- and y-axes <u>must</u> be in reduced form.

Example 2: Determine the equation of the given trigonometric function.



cos curve

amp: $\frac{7-1}{2}$

$= 3$

reflection $-\cos x$

axis: 4

period:

$k = \frac{2\pi}{2\pi}$

$= 1$

no phase shift

$\therefore y = -3\cos x + 4$

sin curve

p.s. right $\frac{\pi}{2}$

$\therefore y = 3\sin\left(x - \frac{\pi}{2}\right) +$

Example 3: A sine function has a maximum value of 9, minimum value of -5, phase shift of $\frac{\pi}{4}$ left, and, period of π. Determine an equation of the function.

max: 9 min: -5

$\therefore$ axis: $\frac{9+(-5)}{2}$

$= 2$

amp: $\frac{9-(-5)}{2}$

$= 7$

p.s.: left $\frac{\pi}{4}$

$k = \frac{2\pi}{\pi}$

$= 2$

$\therefore y = 7\sin\left[2\left(x + \frac{\pi}{4}\right)\right] + 2$