

1. Rearrange the equation to isolate the trig function
2. Use the CAST rule or the Graph to determine all possible solutions within the domain.

Example 1: Solve each of the following,  $0 \leq x \leq 2\pi$ .

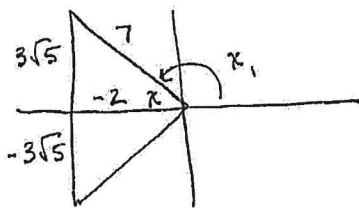
a)  $\cos x = -\frac{2}{7}$

$$y = 7^2 - (-2)^2$$

$$= 49 - 4$$

$$y = \sqrt{45}$$

$$= 3\sqrt{5}$$



$$x = \cos^{-1}\left(+\frac{2}{7}\right) \quad (\text{related acute } \angle)$$

$$\doteq 1.28 \text{ rad} \quad (\text{in radians})$$

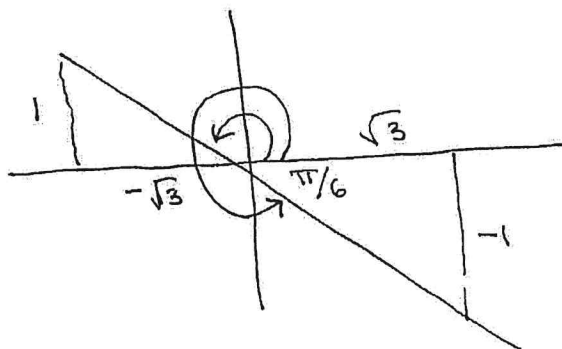
$$x_1 = 3.14 - 1.28 \quad \left. \vphantom{x_1} \right\} \text{Q2}$$

$$\doteq 1.86$$

$$x_2 = 3.14 + 1.28 \quad \left. \vphantom{x_2} \right\} \text{Q3}$$

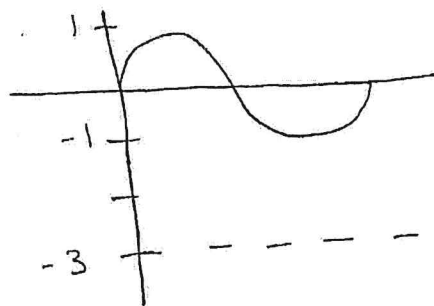
$$= 4.42$$

b)  $\cot x = -\sqrt{3} \Rightarrow \tan x = -\frac{1}{\sqrt{3}}$



$$x_1 = \frac{5\pi}{6} \quad x_2 = \frac{11\pi}{6}$$

c)  $\sin x = -3$



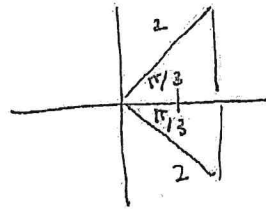
no soln for  $x$ .

Example 2:

Solve  $2\cos\left(x - \frac{\pi}{6}\right) = 1$ ,  $0 \leq x \leq 2\pi$ .

let  $x - \frac{\pi}{6}$  be  $y$

$2\cos y = 1$   
 $\cos y = \frac{1}{2}$



$y = \frac{\pi}{3}$

$y = \frac{5\pi}{3}$

sub back in.

$x - \frac{\pi}{6} = \frac{\pi}{3}$

$x = \frac{2\pi}{6} + \frac{\pi}{6}$

$x = \frac{3\pi}{6}$

$x = \frac{\pi}{2}$

$x - \frac{\pi}{6} = \frac{5\pi}{3}$

$x = \frac{10\pi}{6} + \frac{\pi}{6}$

$x = \frac{11\pi}{6}$

Example 3: Solve  $\sin\theta \tan\theta = \sin\theta$ ,  $0 \leq \theta \leq 2\pi$ .   
 \*cannot just cancel  $\sin$ . Will miss solutions!

$\sin\theta \tan\theta - \sin\theta = 0$

$\sin\theta (\tan\theta - 1) = 0$

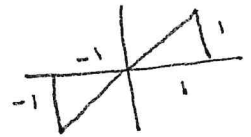
$\sin\theta = 0$

$\therefore \theta = 0, \pi, 2\pi$

$\tan\theta - 1 = 0$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$



Example 4:

Solve for  $x$ ,  $0 \leq x \leq 2\pi$

$3\sin^2 x - \sin x - 2 = 0$

let  $\sin x = y$

$3y^2 - y - 2 = 0$

$3y^2 - 3y + 2y - 2 = 0$

$3y(y-1) + 2(y-1) = 0$

$(3y+2)(y-1) = 0$

$y = -\frac{2}{3}$        $y = 1$

sub back in.

$\sin x = -\frac{2}{3}$

$x_1 = 3.14 + 0.73$

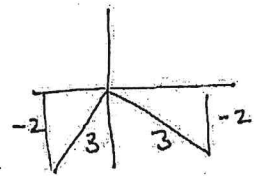
$x_1 = 3.87 \text{ rad}$

$x_2 = 6.28 - 0.73$

$x_2 = 5.55 \text{ rad}$

$\sin x = 1$

$x = \frac{\pi}{2}$



$\beta = \sin^{-1}\left(\frac{2}{3}\right)$   
 $\beta = 0.73$

