

**Example 1:** Solve  $\cos 2x = \frac{1}{2}$  for  $x$ ,  $0 \leq x \leq 2\pi$ .

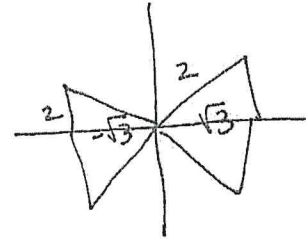
$$2\cos^2 x - 1 = \frac{1}{2}$$

$$2\cos^2 x = \frac{3}{2}$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \sqrt{\frac{3}{4}}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$



$$\beta = \frac{\pi}{6}$$

$$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

**Example 2:** Solve  $-10\cos^2 x - 3\sin x + 9 = 0$  for  $x$ ,  $0 \leq x \leq 2\pi$ .

$$-10(1 - \sin^2 x) - 3\sin x + 9 = 0$$

$$-10 + 10\sin^2 x - 3\sin x + 9 = 0$$

$$10y^2 - 3y - 1 = 0$$

$$10y^2 - 5y + 2y - 1 = 0$$

$$5y(2y - 1) + (2y - 1) = 0$$

$$(2y - 1)(5y + 1) = 0$$

$$\downarrow$$

$$2y = 1$$

$$y = \frac{1}{2}$$

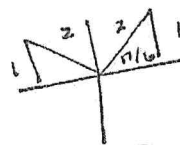
$$\downarrow$$

$$5y = -1$$

$$y = -\frac{1}{5}$$

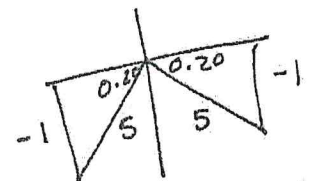
let  $\sin x = y$

$$\sin x = \frac{1}{2}$$



$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin x = -\frac{1}{5}$$



$$x = 3.14 + 0.20$$

$$x = 3.34$$

$$x = 6.28 - 0.20$$

$$x = 6.08$$

**Example 3:** Solve  $\sec^2 x + 5 \tan x = -2$  for  $x$ ,  $0 \leq x \leq 2\pi$ .

$$\tan^2 x + 1 + 5 \tan x + 2 = 0$$

$$\tan^2 x + 5 \tan x + 3 = 0$$

$$y^2 + 5y + 3 = 0$$

not factorable

$$y = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{13}}{2}$$

$$y = \frac{-5 + \sqrt{13}}{2}$$

$$\approx -0.697$$

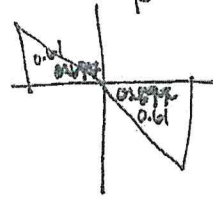
$$y = \frac{-5 - \sqrt{13}}{2}$$

$$\approx -4.30$$

let  $\tan x = y$

$$\tan x = -0.697$$

$$\beta = 0.61$$

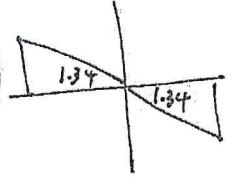


$$x = 3.14 - 0.61 = 2.53$$

$$x = 6.28 - 0.61 = 5.64$$

$$\tan x = -4.30$$

$$\beta = 1.34$$



$$x = 3.14 - 1.34 = 1.80$$

$$x = 6.28 - 1.34 = 4.94$$

✦

**Example 4:** Solve  $2 \sin^2 2x - 1 = -\sin 2x$  for  $x$ ,  $0 \leq x \leq 2\pi$ .

$$2 \sin^2 2\theta + \sin 2\theta - 1 = 0$$

let  $a = \sin 2\theta$

$$2a^2 + a - 1 = 0$$

$$2a^2 + 2a - a - 1 = 0$$

$$2a(a+1) - (a+1) = 0$$

$$(2a-1)(a+1) = 0$$

$$a = \frac{1}{2}$$

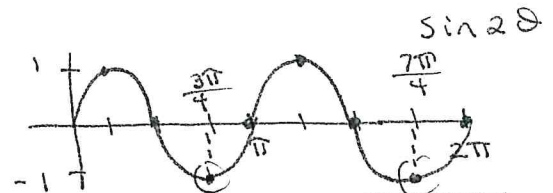
$$a = -1$$

$$\sin 2\theta = \frac{1}{2}$$

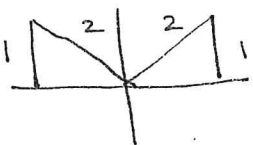
$$\sin 2\theta = -1$$

let  $y = 2\theta$

$$\sin y = \frac{1}{2}$$



$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$



$$\therefore y = \frac{\pi}{6}$$

$$y = \frac{5\pi}{6}$$

$$2\theta = \frac{\pi}{6}$$

$$2\theta = \frac{5\pi}{6}$$

(period)  $+\pi$

$$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$